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TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 929

AERODYNAMICS OF ROTATING-WING AIRCRAFT WITH BLADE-PITCH CONTROL

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Luftfahrtforschung
Vol. 16, No. 7, July 20, 1939
Verlag von R. Oldenbourg, München und Berlin

1.6.1

Washington
February 1940

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AERODYNAMICS OF ROTATING-WING AIRCRAFT WITH BLADE-PITCH CONTROL*

By A. Pflüger

In the studies that have thus far appeared on the aerodynamics of rotating-wing aircraft, rotor systems are investigated of such a character that rotation of the blade about its span axis, except for torsional deflection, is excluded from consideration. In the present report, with the aid of the usual computation methods, a rotor is investigated the pitch of whose blades is capable of being controlled in such a manner that it varies linearly with the flapping angle. To test the effect of this linkage on the aircraft performance, the theory is applied to an illustrative example.

I. PRELIMINARY REMARKS

1. Control Linkage

The aerodynamics of rotating-wing aircraft has been treated in a considerable number of published reports. These are all concerned with wing systems for which the rotor blades, hinge-connected to the axis of rotation, possess two degrees of freedom; a motion of rotation of each blade in the plane of rotation, permitted by the swiveling hinge in the rotor disk (plane containing the hinge and the perpendicular to the rotor axis), and a flapping motion at right angles to the rotor disk permitted by the flapping hinge. With this arrangement, no deflection of the blade occurs about its span axis. The blade pitch, that is, the angle between the zero lift line of the blade section and the plane of rotation (fig. 3) is thus determined by the design and remains constant during the rotation of the wing system.

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*"Aerodynamik des Drehflüglers mit Blattwinkelrücksteuerung." Luftfahrtforschung, vol. 16, no. 7, July 20, 1939, pp. 355-361.

In recent times, a different kinematical system has proven itself practically applicable. With this system, a rotation of the wing about its span axis, in addition to flapping and rotating, is possible. This variation of the blade pitch is, however, positively coupled with the flapping angle in such a manner that the upward motion of the blade reduces the blade pitch. The process can best be understood by considering the design of the rotor system in detail. Such coupling has been practically applied to the Bréguet helicopter (German Patent No. 567,584/62b (1933)) and the Hafner gyroplane (reference 1). Since the latter is described in detail in the literature, the arrangement of its rotor system has been used as the basis for our computation.

The essential characteristic of the design is the so-called "spider" shown in the sketch of figure 1. It consists of the spider axis and the spider arms which are connected with the rotor blades through hinges G and Lever arms H. The rotor blades are attached to the ring R through a Cardan hinge K, making possible the change in blade angle with flapping angle. In the Hafner gyroplane, the flapping hinge is through suitable means placed in the axis of rotation. In figure 1, the more general case of a distance e between the hinge K and the axis is assumed. The spider axis may move up and down in ring R and be inclined in any direction. The possibilities thus provided for the change in the blade pitch are of great importance for the controllability of the aircraft but will not be considered further here. We are interested only in the relation between the flapping and blade angle for steady flight conditions with constant setting of the spider axis with respect to the ring R. This relation may be derived with the aid of figure 1. Let the flapping angle be denoted by β, the blade pitch by ϑ (see figs. 2 and 3); for $\beta = 0$, let $\vartheta = \vartheta_0$. For an upward motion of the blade point A at which the lever arm H is attached to the blade spar, moves up by the amount $\beta(a-e)$. Since the position of the hinge G remains unchanged, the blade pitch is decreased by the amount $\beta \frac{a-e}{h}$. Between ϑ and β, we thus have the linear relation:

$$\vartheta = \vartheta_0 - \frac{a - e}{h} \beta$$

For briefness, we set

$$\vartheta_R = \frac{a - e}{h} \qquad (1)$$

and denote it as the linkage ratio. We thus have

$$\vartheta = \vartheta_0 - \vartheta_R \beta \qquad (2)$$

With regard to the order of magnitude of ϑ_R, the following may be said. If the flapping hinge K is situated in the axis of rotation $\vartheta_R = a/h$: i.e., equal to cotangent of the angle formed by the blade axis with the line joining the hinge G and the center of the rotor system. For reasons of symmetry, this angle will be chosen preferably about 60°. The linkage ratio will then have a value of $\vartheta_R = 0.577$. If e is greater than 0, a smaller value of ϑ_R will be obtained; for $e = a$, $\vartheta_R = 0$ and we obtain the usual system of the Cierva autogiro. In general, e should be as small as possible, so that in practical designs it will lie between the values 0.3 and 0.6.

2. Statement of the Problem and Symbols Used

The object of the following computation is to extend the results already obtained on the aerodynamics of rotating-wing aircraft in steady, forward flight to the case where we have the above-described linkage between the blade and flapping angles.

The papers on gyroplanes are all based on the original investigations of Glauert and Lock. Following upon the work of those men, a considerable number of further investigations have been conducted of which chief mention is to be made of a paper by Wheatley (reference 2) in which the entire theory of the computation of the air forces of rotating-wing aircraft is built up anew and the old computations are considerably extended. There is also to be mentioned a paper by Sissingh (reference 3) which follows closely the work of Wheatley but introduces further refinements, so that it is possible to estimate the importance of certain of the assumptions of Wheatley by a more accurate computation. In Sissingh's paper will be found a detailed bibliography, so we shall here dispense with it.

The computation given below largely follows the work of Wheatley. We may therefore dispense with a detailed explanation of the assumptions and formulas which hold for all rotating-wing aircraft independent of any particular kinematics of the wing system. The investigation is restricted to the simplest case of untwisted rectangular blades: i.e., the blade pitch ϑ_0 and the blade chord are constant. The blade twist due to torsional moments about the blade-span axis is by suitable constructive means (smallest possible distance between blade center of gravity, shear center of blade cross section, and center of pressure of blade section) held low enough so that it may be neglected.

The following notation is used:

R, rotor radius (distance of blade tip from rotation axis);

z, number of rotor blades;

t, blade chord;

$\sigma = \frac{zt}{\pi R}$, solidity;

V, forward velocity;

ω, angular velocity of the rotor blades;

w, induced velocity;

α, angle of attack of the rotor disk.

Then

$V \cos \alpha$ is the component of the forward velocity in the rotor disk;

$V \sin \alpha - w$, component of the forward velocity normal to the rotor disk (fig. 2).

From these, we further define the nondimensional coefficients:

$$\mu = \frac{V \cos \alpha}{\omega R},$$

$$\lambda = \frac{V \sin \alpha - w}{\omega R}$$

A, rotor lift;

W, rotor drag (not of the entire aircraft);

S, rotor thrust (component of the resultant air force in the direction of the axis);

For the purposes of our investigation, we may with sufficient accuracy set S equal to the resultant air force, so that $A = S \cos \alpha$;

M_d, applied moment of the rotor blades about the axis of rotation (for the autogiro $M_d = 0$);

$k_S = \dfrac{S}{\frac{1}{2} \rho \omega^2 R^4 \pi}$, thrust coefficient;

$\epsilon = \dfrac{W}{A}$, lift-drag ratio;

The above-defined magnitudes refer to the rotating-wing system as a whole. For the flow and lift relations at each blade element, the following symbols are used. (See fig. 3.)

r, distance of a blade element from the axis of rotation;

$x = \dfrac{r}{R}$;

m, blade mass per unit length along blade span;

$J = \int_0^R m\, r^2 dr$, mass moment of inertia of a blade about the point $r = 0$;

$M_g = \int_0^R m\, g\, r\, dr$, moment of the blade weight about the point $r = 0$;

α_r, angle between zero lift line and resultant flow at blade element at position r;

c_a, blade section lift coefficient;

c_a', constant mean value of slope of curve $c_a = f(\alpha_r)$;

$\overline{c_w}$, constant mean value of drag coefficient c_w of blade section;

$u_p \omega R$, velocity at blade element parallel to rotor axis;

$u_t \omega R$, velocity at blade element in the rotor disk;

$\varphi = \frac{u_t}{u_p}$, angle between direction of resultant flow and rotor disk;

ψ, azimuth angle of blades measured in plane of rotor (fig. 2);

$\gamma = \frac{\rho t R^4 c_a'}{J}$, blade mass constant;

M_s, moment of air forces of a blade about the flapping hinge;

B, factor to take account of the tip losses.

The ratio μ serves in the computation as the independent variable. The object of the investigation is to determine the flapping angle β and the magnitudes k_s, α, and ϵ.

II. THE AIRCRAFT IN STEADY FORWARD FLIGHT

1. Induced Velocity and Angle of Attack

In determining the induced velocity and the angle of attack, the rotor system is considered as a lifting vortex of width 2R. Under the usual assumption that the induced velocity w and hence also the ratio λ are constant for the entire blade-swept area, we have, according to Wheatley:

$$w = \frac{k_s}{4\sqrt{u^2 + \lambda^2}} \omega R \tag{3}$$

and

$$\tan \alpha = \frac{\lambda}{\mu} + \frac{k_s}{4\mu\sqrt{\mu^2 + \lambda^2}} \tag{4}$$

k_s and λ are for the present unknown magnitudes and will be determined below as functions of μ. (It is to be noted that between the thrust coefficient k_s and the magnitude C_T employed by Wheatley there is the relation $k_s = 2C_T$)

2. Flow Velocities at Blade Element

For the velocity at the blade element in the plane of the rotor, we obtain

$$u_t \ \omega R = \omega r + \mu \omega R \sin \psi$$

$$u_t = x + \mu \sin \psi \tag{5}$$

and for the velocity parallel to the rotor axis

$$u_p \, \omega R = \lambda \, \omega R - r \frac{d\beta}{dt} - \mu \omega R \, \beta \cos \psi$$

$$u_p = \lambda - \frac{x}{\omega} \frac{d\beta}{dt} - \mu \, \beta \cos \psi \tag{6}$$

where $\frac{d\beta}{dt}$ is the derivative of the flapping angle β with respect to time. The angle β (acute angle between the blade span axis and the rotor disk) is expressed in terms of the azimuth angle ψ by a Fourier series in which terms only up to the second harmonic are retained:

$$\beta = a_0 - a_1 \cos \psi - b_1 \sin \psi - a_2 \cos 2\psi - b_2 \sin 2\psi \tag{7}$$

There is then obtained from (6), with $\psi = \omega t$

$$\begin{aligned} u_p = {} & \lambda + \frac{1}{2} \mu a_1 + \left(- \mu a_0 + x b_1 + \frac{1}{2} \mu a_2\right) \cos \psi \\ & + \left(- x a_1 + \frac{1}{2} \mu b_2\right) \sin \psi + \left(\frac{1}{2} \mu a_1 + 2 x b_2\right) \cos 2\psi \\ & + \frac{1}{2} \mu b_1 - 2 x a_2 \quad \sin 2\psi \\ & \qquad + \frac{1}{2} \mu a_2 \cos 3\psi + \frac{1}{2} \mu b_2 \sin 3\psi \end{aligned} \tag{8}$$

The radial velocity in the direction of the blade span axis need not be taken into account. Its effect on the magnitudes to be investigated will as usual be neglected.

3. Flapping Angle

The flapping motion β is obtained by determining the Fourier coefficient a_0, a_1, b_1, a_2, b_2. In most of the investigations on the aerodynamics of helicopters and autogiros, the second harmonics a_2 and b_2, in contrast to the procedure of Wheatley, are dropped, being considered negligible. This is justified by experience as long as we are mainly concerned with the aerodynamic behavior of the rotating-wing system in its total effect: i.e., in the magnitudes S, A, W, ω, V. For the designer, it is important, however, in order to estimate the strength of the rotor blades, to know as accurately as possible the air forces at each blade element which forces are considerably affected by the second harmonic of the flapping motion. The coefficients a_2 and b_2 are therefore in the following retained.

The differential equation of the flapping motion is obtained from the moment equilibrium equation about the flapping hinge. In this condition there enter the air forces, the inertia forces of the flapping blade, the centrifugal forces, and the weight. With the notation given above, there is obtained the known relation

$$J\,\omega^2\left(\frac{d^2\beta}{d\,\psi^2} + \beta\right) = M_S - M_g \tag{9}$$

It is here assumed that the distance e between the flapping hinge and the axis of rotation is zero or small enough so that its effect may be neglected.

The moment of the air forces about the flapping hinge, under the assumption that the resultant velocity at the blade element is with sufficient accuracy given by $u_t\,\omega\,R$, is

$$M_S = \int_0^{BR} \frac{1}{2}\,\rho\,t\,u_t^2\,\omega^2\,R^2\,c_a\,r\,d\,r$$

$$M_S = \frac{1}{2}\,\rho\,t\,\omega^2\,R^4 \int_0^{B} u_t^2\,c_a\,x\,d\,x \tag{10}$$

To take account of the thrust losses at the blade tip, we integrate, following Wheatley, not up to R but only up to BR, where

$$B = 1 - \frac{t}{2\,R}$$

Now

$$c_a = c_a'\,\alpha_r$$

and from figure 3

$$\alpha_r = \vartheta + \varphi$$

With the aid of (2), there is then obtained

$$\alpha_r = \vartheta_0 - \vartheta_R\beta + \frac{u_p}{u_t} \tag{11}$$

This holds, however, only for the advancing blade section. For the retreating blade ($\pi < \psi < 2\,\pi$ and $0 < r < -\mu\, r \sin\psi$), α_r is as usual replaced by $-\alpha_r$ and it is assumed that the slope c_a' of the lift curve has approximately the same value as for the advancing blade. We set

$$M_S = M_S^{\,I} + M_S^{\,II}$$

where $M_S^{\,I}$ is the air-force moment that is obtained by substituting the angle α_r valid for the forward moving blade, and $M_S^{\,II}$ is the correction term that takes the retreating motion into account. From (10), there is obtained

$$\left.\begin{aligned} M_S^{\mathrm{I}} &= \frac{1}{2}\varrho\, t\,\omega^2 R^4 c_a' \int_0^B [(\vartheta_0 - \vartheta_R \beta)\, u_t^2 + u_t u_p]\, x\, dx \\ M_S^{\mathrm{II}} &\begin{cases} = 0 \text{ for } 0 < \psi < \pi \\ = -\varrho\, t\,\omega^2 R^4 c_a' \int_0^{-\mu \sin\psi} [(\vartheta_0 - \vartheta_R \beta)\, u_t^2 + u_t u_p]\, x\, dx \\ \text{for } \pi < \psi < 2\pi \end{cases} \end{aligned}\right\} \quad (12\text{ a, b})$$

M_S is now developed into a Fourier series of which again only terms up to the second harmonic are retained:

$$M_S = \frac{1}{2}\varrho\, t\,\omega^2 R^4 c_a' (A_0 + A_1 \cos\psi + B_1 \sin\psi + A_2 \cos 2\psi + B_2 \sin 2\psi) \quad (13)$$

For each of the harmonics, if the small effect of the retreating motion is taken into account only in the coefficients A_0, A_1, and B_1, the following expressions are valid:

$$\left.\begin{aligned} A_0 &= \frac{1}{2\pi}\int_0^{2\pi} \frac{M_S^{\mathrm{I}}}{\frac{1}{2}\varrho\, t\,\omega^2 R^4 c_a'}\, d\psi + \frac{1}{2\pi}\int_\pi^{2\pi} \frac{M_S^{\mathrm{II}}}{\frac{1}{2}\varrho\, t\,\omega^2 R^4 c_a'}\, d\psi, \\ A_1, B_1 &= \frac{1}{\pi}\int_0^{2\pi} \frac{M_S^{\mathrm{I}}(\cos\psi, \sin\psi)}{\frac{1}{2}\varrho\, t\,\omega^2 R^4 c_a'}\, d\psi + \frac{1}{\pi}\int_\pi^{2\pi} \frac{M_S^{\mathrm{II}}(\cos\psi, \sin\psi)}{\frac{1}{2}\varrho\, t\,\omega^2 R^4 c_a'}\, d\psi, \\ A_2, B_2 &= \frac{1}{\pi}\int_0^{2\pi} \frac{M_S^{\mathrm{I}}(\cos 2\psi, \sin 2\psi)}{\frac{1}{2}\varrho\, t\,\omega^2 R^4 c_a'}\, d\psi \end{aligned}\right\} \quad (14\text{ a—c})$$

Substituting (5), (8), and (7) in (12) and then in (14), there is obtained, after carrying out the integrations

$$\left.\begin{aligned} A_0 &= \frac{1}{3}\lambda B^3 + \frac{2}{9\pi}\lambda \mu^3 + \frac{1}{8}\mu^2 B^2 b_2 \\ &\quad + \vartheta_0 \frac{1}{4}\left(B^4 + \mu^2 B^2 - \frac{1}{8}\mu^4\right) \\ &\quad - \vartheta_R \frac{1}{4}\left(B^4 a_0 + \mu^2 B^2 a_0 - \frac{1}{8}\mu^4 a_0 \right. \\ &\quad \left. - \frac{4}{3}\mu B^3 b_1 + \frac{1}{2}\mu^2 B^2 a_2\right), \\ A_1 &= -\frac{1}{3}\mu B^3 a_0 - \frac{4}{45\pi}\mu^4 a_0 + \frac{1}{4} B^4 b_1 \\ &\quad + \frac{1}{8}\mu^2 B^2 b_1 - \frac{1}{6}\mu B^3 a_2 \\ &\quad + \vartheta_R \frac{1}{4}\left(B^4 a_1 + \frac{1}{2}\mu^2 B^2 a_1 + \frac{4}{3}\mu B^3 b_2\right), \\ B_1 &= \frac{1}{2}\lambda \mu B^2 - \frac{1}{8}\lambda \mu^3 - \frac{1}{4} B^4 a_1 \\ &\quad + \frac{1}{8}\mu^2 B^2 a_1 - \frac{1}{6}\mu B^3 b_2 \\ &\quad + \vartheta_0 \left(\frac{2}{3}\mu B^3 + \frac{8}{45\pi}\mu^4\right) \\ &\quad - \vartheta_R \left(\frac{2}{3}\mu B^3 a_0 + \frac{8}{45\pi}\mu^4 a_0 \right. \\ &\quad \left. - \frac{1}{4} B^4 b_1 - \frac{3}{8}\mu^2 B^2 b_1 + \frac{1}{3}\mu B^3 a_2\right), \\ A_2 &= \frac{1}{3}\mu B^3 a_1 + \frac{1}{2} B^4 b_2 - \frac{1}{4}\mu^2 B^2 \vartheta_0 \\ &\quad + \vartheta_R \frac{1}{4}\left(\mu^2 B^2 a_0 - \frac{4}{3}\mu B^3 b_1 \right. \\ &\quad \left. + B^4 a_2 + \mu^2 B^2 a^2\right), \end{aligned}\right\} \quad (15\text{ a—d})$$

$$\left.\begin{aligned} B_2 &= -\frac{1}{4}\mu^2 B^2 a_0 + \frac{1}{3}\mu B^3 b_1 - \frac{1}{2} B^4 a_2 \\ &\quad + \vartheta_R \frac{1}{4}\left(\frac{4}{3}\mu B^3 a_1 + B^4 b_2 + \mu^2 B^2 b_2\right) \end{aligned}\right\} \quad (15\text{ e})$$

With the aid of (7), there is obtained from (9)

$$M_S = \frac{1}{2}\varrho\, t\,\omega^2 R^4 c_a' \left[\frac{2}{\gamma}\left(a_0 + 3 a^2 \cos 2\psi + 3 b_2 \sin 2\psi - \frac{2 M_g}{\gamma J \omega^2}\right)\right] \quad (16)$$

with $\gamma = \frac{\varrho\, t\, R^4 c_a'}{J}$

By comparing the coefficients of the trigonometric functions of equations (13) and (16), the following relations are obtained.

$$\left.\begin{aligned} \frac{2}{\gamma} a_0 + \frac{2 M_g}{\gamma J \omega^2} &= A_0, \\ A_1 &= 0, \\ B_1 &= 0, \\ \frac{6}{\gamma} a_2 &= A_2, \\ \frac{6}{\gamma} b_2 &= B_2 \end{aligned}\right\} \quad (17\text{ a—e})$$

Equations (17) in connection with (15) are now to be solved for the Fourier coefficients of the flapping motion. It is sufficient here to compute a_0, a_1, b_1 up to terms of the order of magnitude μ^4 and a_2, b_2 up to terms of the order of μ^2. Under this assumption a_2 and b_2 may be determined from equations (17 d,e) if, in the expressions for A_2 and B_2, the following approximation values are substituted for the coefficients a_0, a_1, b_1

$$\left.\begin{aligned} a_0 &\sim \frac{\gamma}{8 + \gamma \vartheta_R B^4}\left(\frac{4}{3}\lambda B^3 + \vartheta_0 B^4\right), \\ a_1 &\sim \frac{2\mu}{B^2(1 + \vartheta_R^2)}\left(\lambda + \frac{4}{3} B \vartheta_0 - \frac{2}{3} B \vartheta_R a_0\right), \\ b_1 &\sim \frac{4}{3}\mu \frac{a_0}{B} - \vartheta_R a_1 \end{aligned}\right\} \quad (18\text{ a—c})$$

These relations are obtained from (17), if in (17a) for the computation of a_0 only terms without μ are retained and in (17 b,c) for computing a_1, b_1 only terms with μ^1. With the aid of (18), there is then obtained from (17 d,e)

$$\left.\begin{aligned} b_2 &= \frac{B^5 \mu^2}{72 + 9\gamma \vartheta_R B^4} \\ &\quad \times \frac{\vartheta_R \lambda \gamma B^4 - 40\lambda - 50 \vartheta_0 B - \frac{7}{4}\vartheta_0 \vartheta_R \gamma B^5}{B^8 + \left(\frac{12}{\gamma} - \frac{1}{2} B^4 \vartheta_R\right)^2} \\ a_2 &= \frac{7\gamma \mu^2 B}{18(8 + \gamma \vartheta_1 B^4)} \\ &\quad \times \left(\frac{4}{3}\lambda + \vartheta_0 B\right) - \frac{b_2}{B^4}\left(\frac{12}{\gamma} - \frac{1}{2} B \vartheta_R\right) \end{aligned}\right\} \quad (19\text{ a, b})$$

Knowing a_2 and b_2, the remaining coefficients of the flapping motion may be determined from (17 a-c) up to the order of magnitude of μ^4. Carrying out the computation, there is obtained

$$\left.\begin{aligned} a_0 &= \frac{1}{\Phi_1\left(\frac{2}{\gamma} + \Phi_2 \vartheta_R\right) - \frac{4}{9}\mu^2 B^2 \vartheta_R^2 \left[B^3\left(\frac{1}{\vartheta_R} + 2\vartheta_R^2\right) - 2\mu^2\right]} \\ &\quad \times \left\{\Phi_1\left[\frac{1}{3}\lambda B^3 + \frac{2}{9\pi}\lambda \mu^3 - \frac{2 M_g}{\gamma J \omega^2} + \frac{1}{8}\mu^2 B^2 (b_2 - \vartheta_R a_2) \right.\right. \\ &\quad \left. + \Phi_2 \vartheta_0\right] - \frac{2}{9}\mu^2 \vartheta_R^2 B \left[3\lambda B^2 + \frac{9}{4}\lambda \mu^2 + 4\vartheta_0 B (B^2 + \mu^2) \right. \\ &\quad \left.\left. - B^3\left(\frac{1}{\vartheta_R} + 2\vartheta_R\right) a_2 + B^3 b_2\right]\right\} \end{aligned}\right\} \quad (19\text{ c})$$

with
$$\left.\begin{aligned}
\Phi_1 &= \left(B^2+\frac{1}{2}\mu^2\right)(1+\vartheta_R^2)+2\mu^2\vartheta_R,\\
\Phi_2 &= \frac{1}{4}\left(B^4+B^2\mu^2-\frac{1}{8}\mu^4\right),\\
a_1 &= \frac{\mu}{\frac{1}{2}B^4(1+\vartheta_R^2)+B^2\mu^2\vartheta_R}\\
&\quad\times\left[\lambda B^2+\frac{1}{4}\lambda\mu^2+\vartheta_0\left(\frac{4}{3}B^3+\frac{2}{3}\mu^2 B+\frac{16}{45\pi}\mu^3\right)-\vartheta_R\left(\frac{2}{3}B^3a_0-\frac{1}{3}\mu^2 B a_0+\frac{8}{45\pi}\mu^3 a_0+\frac{1}{3}B^3 a_2\right)-\frac{1}{3}B^3 b_2(1+2\vartheta_R^2)\right],\\
b_1 &= \frac{4\mu}{B^4+\frac{1}{2}\mu^2B^2}\\
&\quad\times\left(\frac{1}{3}B^3a_0+\frac{4}{45\pi}\mu^3a_0+\frac{1}{6}B^3a_2-\frac{1}{3}\vartheta_R B^3 b_2\right)-a_1\vartheta_R
\end{aligned}\right\}\quad(19\text{d})$$

4. Thrust Coefficient

For the total thrust of all of the z blades, there is obtained for one revolution the mean value

$$S=\frac{z}{2\pi}\int_0^{2\pi}\int_0^{BR}\frac{1}{2}\varrho\, t\, u_t^2\,\omega^2 R^2 c_a\, d r\, d\psi.$$

For the thrust coefficient, there is obtained

$$k_S=\frac{S}{\frac{1}{2}\varrho\,\omega^2R^4\pi}=\frac{\sigma}{2\pi}\int_0^{2\pi}\int_0^{B}u_t^2 c_a\,d x\,d\psi.$$

where $\sigma=\frac{z t}{2\pi}$

Substituting the angle of attack α_r and subdividing the range of integration for advancing and retreating motion, there is obtained.

$$\left.\begin{aligned}
k_S=\frac{\sigma c_a'}{2\pi}\Bigg\{&\int_0^{2\pi}\int_0^{B}[(\vartheta_0-\vartheta_R\beta)u_t^2+u_t u_p]\,dx\,d\psi\\
&-2\int_0^{2\pi}\int_0^{-\mu\sin\psi}[(\vartheta_0-\vartheta_R\beta)u_t^2+u_t u_p]\,dx\,d\psi\Bigg\}
\end{aligned}\right\}\quad(20)$$

With the aid of (5), (8), and (7), there is obtained from (20) after evaluating the integrals up to terms of the order of magnitude μ^4

$$\left.\begin{aligned}
k_S=\sigma c_a'\Bigg\{&\frac{1}{2}\lambda\left(B^2+\frac{1}{2}\mu^2\right)+\vartheta_0\frac{1}{3}\left(B^3+\frac{3}{2}\mu^2B-\frac{4}{3\pi}\mu^3\right)\\
&+\frac{1}{8}\mu^3a_1+\frac{1}{4}\mu^2Bb_2-\vartheta_R\left[\frac{1}{3}a_0\left(B^3+\frac{2}{3}\mu^2B-\frac{4}{3\pi}\mu^3\right)-\frac{1}{2}\mu b_1\left(B^2+\frac{1}{4}\mu^2\right)+\frac{1}{4}\mu^2Ba_2\right]\Bigg\}
\end{aligned}\right\}\quad(21)$$

5. Drag-Lift Coefficient

To determine the drag and power of the rotor a knowledge of the drag-lift ratio ϵ is required, the latter being computed from the energy losses of the rotating-wing system. These losses arise from the induced velocity and from the drags at each blade element in rotating. The first-named portion of the losses is obtained from considerations on the rotor system in its total effect, the corresponding formulas not being effected by the linkage between the pitch and flapping angles. The linkage ratio ϑ_R does not enter into the second portion of the losses because a mean value of c_w independent of the angle of attack is used in determining the drag of a blade element. Wheatley's value of the lift-drag ratio can therefore be taken over without modification, his expression being

$$\epsilon=\frac{\bar{c}_w\sigma}{4\mu k_s}\left(1+3\mu^2+\frac{3}{8}\mu^4\right)+\frac{k_s}{4\mu\sqrt{\mu^2+\lambda^2}}\quad(22)$$

6. Balance of Moments about the Vertical Hinge

All of the magnitudes thus far determined contain the nondimensional ratio λ whose dependence on μ is still to be determined. A relation between λ and μ is obtained from the condition of moment equilibrium about the vertical hinge. This condition is

$$M_d+\frac{z}{2\pi}\int_0^{2\pi}\int_0^{BR}\frac{1}{2}\varrho\, t\, u_t^2\omega^2R^2c_a\varphi\, r\,dr\,d\psi-\frac{z}{2\pi}\int_0^{2\pi}\int_0^{R}\frac{1}{2}\varrho\, t\, u_t^2\omega^2R^2\bar{c}_w\, r\,dr\,d\psi=0$$

or

$$\frac{M_d}{\frac{1}{4}\varrho\,\omega^2R^5\sigma}+\int_0^{2\pi}\int_0^{B}u_t^2c_a\varphi\, x\,dx\,d\psi-\int_0^{2\pi}\int_0^{1}u_t^2\bar{c}_w\, x\,dx\,d\psi=0.$$

The first of the double integrals represents the contribution of the lift to the moment, the second, the contribution of the drag. The latter integral is to be integrated from 0 to 1, not only to B because the drag is not decreased by the thrust drop at the blade tips. Taking account of the retreating motion of the blades, there is obtained after substituting the expressions for c_a and φ

$$\left.\begin{aligned}
&\frac{M_d}{\frac{1}{4}\varrho\,\omega^2R^5\sigma c_a'}+\int_0^{2\pi}\int_0^{B}[(\vartheta_0-\vartheta_R\beta)u_pu_t+u_p^2]\,x\,dx\,d\psi\\
&-2\int_0^{2\pi}\int_0^{-\mu\sin\psi}[(\vartheta_0-\vartheta_R\beta)u_pu_t+u_p^2]\,x\,dx\,d\psi\\
&-\frac{\bar{c}_w}{c_a'}\int_0^{2\pi}\int_0^{1}u_t^2x\,dx\,d\psi+2\frac{\bar{c}_w}{c_a'}\int_0^{2\pi}\int_0^{-\mu\sin\psi}u_t^2x\,dx\,d\psi
\end{aligned}\right\}\quad(23)$$

Performing the integrations and rearranging the terms, there is finally obtained

$$\left.\begin{aligned}
&\lambda^2\left(B^2-\frac{1}{2}\mu^2\right)+\lambda\Bigg\{\mu a_1\left(B^2-\frac{3}{4}\mu^2\right)+\vartheta_0\left(\frac{2}{3}B^3-\frac{4}{9\pi}\mu^3\right)\\
&\quad-\vartheta_R\left[a_0\left(\frac{2}{3}B^3+\frac{4}{9\pi}\mu^3\right)-\frac{1}{2}\mu b_1\left(B^2-\frac{1}{4}\mu^2\right)\right]\Bigg\}\\
&\quad-\frac{2}{3}\mu B^3a_0b_1+\frac{1}{2}\mu^2B^2\left(a_0^2+\frac{1}{2}a_1^2-a_0a_2\right)\\
&\quad-\frac{1}{8}\mu^4a_0^2+\frac{1}{4}B^2\left(B^2+\frac{1}{2}\mu^2\right)(a_1^2+b_1^2)\\
&\quad+B^4(a_2^2+b_2^2)+\frac{1}{3}\mu B^3(a_1b_2-b_1a_2)+\frac{1}{4}\vartheta_0\mu^2B^2b_2\\
&\quad-\vartheta_R\left[\frac{1}{3}\mu B^3a_0a_1+\frac{1}{2}\mu^2B^2\left(a_0b_2-\frac{1}{2}a_1b_1\right)-\frac{1}{6}B^3\mu(a_1a_2+b_1b_2)\right]-\frac{\bar{c}_w}{2c_a'}\left(1+\mu^2-\frac{1}{8}\mu^4\right)\\
&\quad+\frac{M_d}{\frac{1}{4}\varrho\,\omega^2R^5\pi\sigma c_a'}=0
\end{aligned}\right\}\quad(24)$$

The practical computation of the ratio λ as a function of μ is somewhat difficult since (24) contains the coefficients of the flapping motion, which coefficients, according to (19), again depend on λ and μ. If λ is known, however, no difficulty is encountered in determining β, α, k_s, and ϵ.

III. ILLUSTRATIVE EXAMPLE

The formulas set up in the preceding section will now be applied to a numerical example. The object of the computation is not so much to clarify the process of practical computations as to obtain a numerical estimate of the significance of the pitch and flapping angle linkage on the aircraft performance.

As an example, there will be chosen an autogiro of the usual present-day design. The initial values used as a basis for the computation and which approximately correspond in their order of magnitude to the design factors used for the C 30 type autogiro are the following:

$G = 900$ kg	$M_d = 0$
$R = 6.00$ m	$c_a{}' = 5.6$
$z = 3$	$c_w = 0.014$
$t = 0.28$ m	$\vartheta_o = 6^\circ$
$J = 25.9$ mkg s^2	$\vartheta_R = 0.45$
	$B = 0.98$

With these values, there is obtained from (19) in connection with (24) the variation of the Fourier coefficients of the flapping motion with μ shown in figure 4. In figure 5, k_s is plotted from (21) and in figure 6, the lift-drag ratio ϵ from (22). For $A = G$,

$$S = \frac{A}{\cos \alpha}$$

the rotational speed $n = \frac{30}{\pi}\omega$ is obtained from

$$k_s = \frac{S}{\frac{1}{2}\,\rho\,\omega^2\,R^4\,\pi}$$

$$n = \frac{30}{\pi}\sqrt{\frac{2\,G}{k_s\,\rho\,\pi\,R^4\,\cos\alpha}}$$

The result is plotted in figure 7 for $G = 900$ kg. The dependence of the forward speed on μ, which is obtained from the relation

$$V = \frac{\mu\,\omega\,R}{\cos\alpha}$$

is shown on figure 8. The maximum velocity of the aircraft will lie near the value $\mu = 0.4$ ($V = 208$ km/h). For this flight condition, figure 9 shows the corresponding flapping motion.

The coefficient a_0, denoted as the coning angle, has the value 4.73°. To a flapping angle of this magnitude, there corresponds the blade pitch angle

$$\begin{aligned}\vartheta_\beta = a_0 &= \vartheta_0 - \vartheta_R\,a_0\\ &= 6 - 0.45\times 4.73\\ &= 3.87^\circ\end{aligned}$$

The rotor thus operates on the average with a pitch of the usual order of magnitude of about 4°.

The important result brought out in figures 5 to 9 is that the autogiro investigated having the blade pitch control described does not show any particular, unexpected properties but behaves entirely like a normal autogiro of the Cierva type. (Compare the computations of Wheatley, N.A.C.A. Reports Nos. 487 and 591.) Particularly noteworthy is the fact that the amplitude of the flapping motion, i.e., chiefly the coefficient a_1, is of the usual magnitude, so that the type of linkage described does not, as might have been expected, lead to a decrease in the flapping motion.

All this leads to the final conclusion that the kinematics of the blade-pitch control of the autogiro, while offering no disadvantages, does not offer any particular advantages as far as behavior in steady forward flight is concerned. It is true that in flight under gust conditions the linkage gives the rotor less sensitivity to the gust. The use of the linkage will hardly prove worthwhile, however, unless other advantages are gained through its application. The advantages of the spider construction described in section I are to be found chiefly in the control possibilities of the rotating-wing system, because of the fact that the spider axis may be displaced up and down or tilted to the side in the ring R. The effect of the linkage is to be considered a by-product conditioned by the design which, however, does not impair the aerodynamic behavior of the aircraft. It should be pointed out, however, that caution must be employed in generalizing the results obtained from the computation example to linkage ratios that considerably exceed the value of $\vartheta_R = 0.45$ investigated.

IV. SUMMARY

The method developed by Wheatley for the computation of the air forces of rotating-wing aircraft is extended in that instead of a constant-blade-pitch angle a linear dependence is assumed between that angle and the flapping angle. The application of the formulas obtained to an autogiro with a linkage ratio of the usual magnitude shows that for steady, forward flight no particular effects arise on the aerodynamic behavior of the aircraft.

Translation by S. Reiss,
National Advisory Committee
for Aeronautics.

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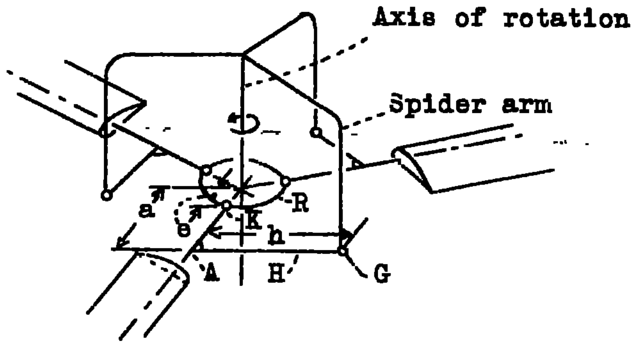

Figure 1.- Autogiro rotor with linkage between the blade pitch angle and the flapping angle.

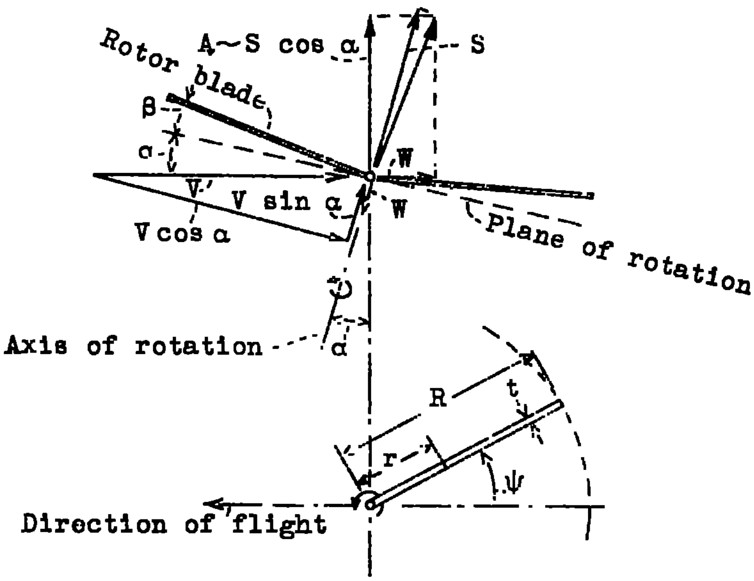

Figure 2.- Velocities and forces of the rotor system.

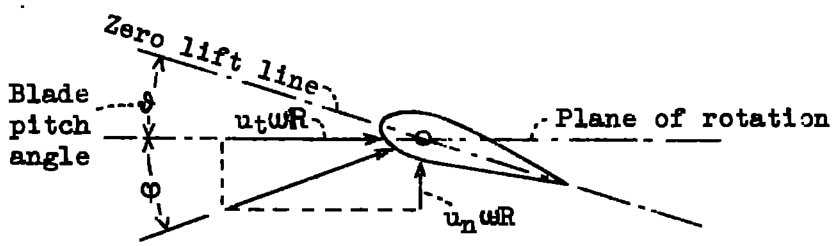

Figure 3.- Velocities at the blade element.

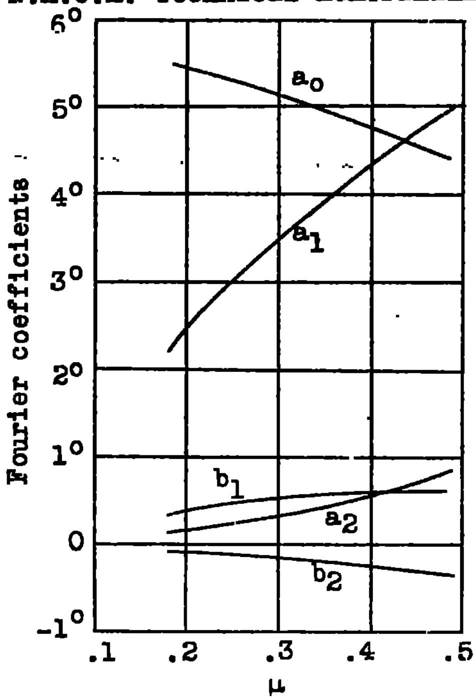

Figure 4.- Fourier coefficients of the flapping motion as a function of the μ.

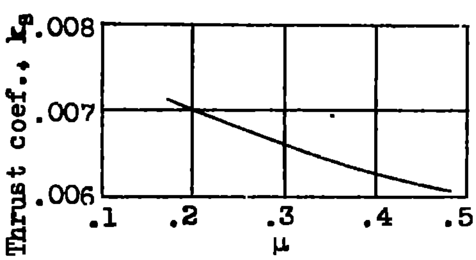

Figure 5.- Thrust coefficient as a function of μ.

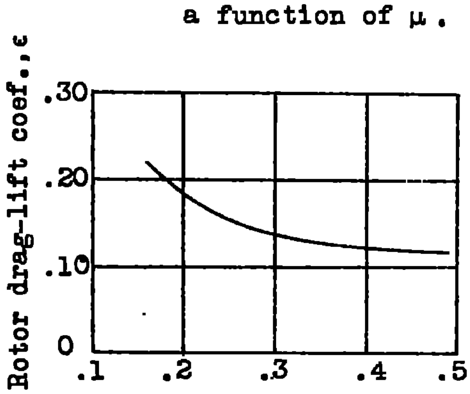

Figure 6.- Rotor drag-lift coefficient as a function of μ.

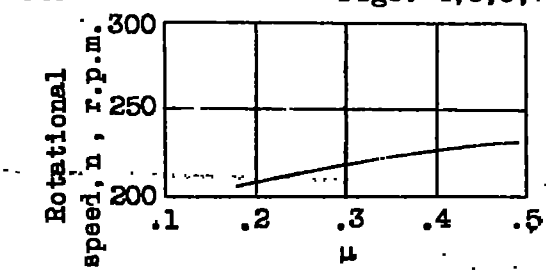

Figure 7.- Rotational speed as a function of μ for G = 900 kg.

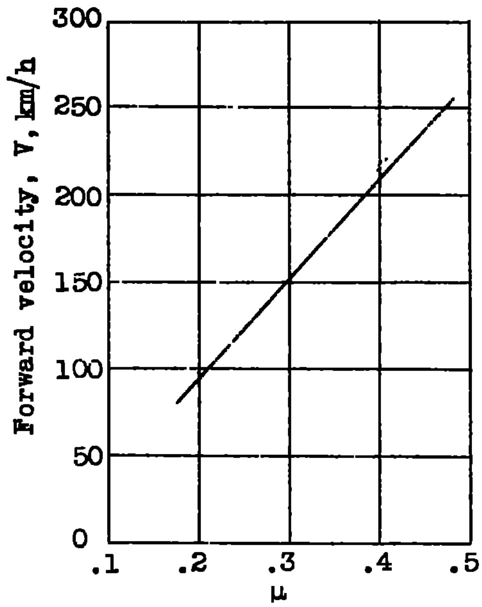

Figure 8.- Forward velocity as a function of μ for G = 900 kg.

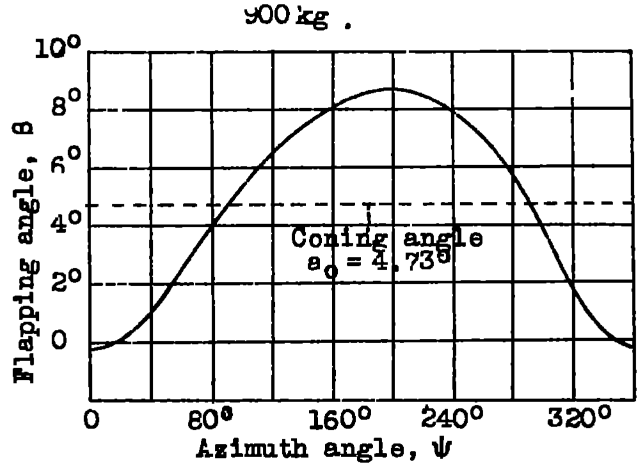

Figure 9.- Flapping angle as a function of the azimuth angle for G = 900 kg and μ = 0.4.